

Klein tunneling of light in fiber Bragg gratings

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Abstract

A photonic analogue of Klein tunneling (KT), i.e. of the exotic property of relativistic electrons to pass a large repulsive and sharp potential step, is proposed for pulse propagation in a nonuniform fiber Bragg grating with an embedded chirped region. KT can be simply observed as the opening of a transmission window inside the grating stop band, provided that the impressed chirp is realized over a length of the order of the analogue of the Compton wavelength.

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I. INTRODUCTION

A remarkable prediction of the Dirac equation is that a below-barrier electron can pass a large repulsive and *sharp* potential step without the exponential damping expected for a non-relativistic particle. Such a transparency effect, originally predicted by Klein [1] and referred to as Klein tunneling (KT), arises from the existence of negative-energy solutions of the Dirac equation and requires a potential step height ΔV of the order of twice the rest energy mc^2 of the electron [2]. Relativistic tunneling across a *smooth* potential step, which describes the more physical situation of a constant electric field E in a finite region of space of length l , was subsequently studied by Sauter [3]. Sauter showed that to observe barrier transparency the potential increase $\Delta V \simeq eEl$ should occur over a distance l of the order or smaller than the Compton wavelength $\lambda_C = \hbar/(mc)$, the transmission probability rapidly decaying toward zero for a smoother potential increase [2–4]. The required field corresponds to the critical field for e^+e^- pair production in vacuum, and its value is extremely strong making the observation of relativistic KT for electrons very challenging. Therefore, growing efforts have been devoted to find experimentally accessible systems to investigate analogs of relativistic KT [5]. Recently, great interest has suscitated the proposal [6] and first experimental evidences [7, 8] of KT for non-relativistic electrons in graphene, which behave like massless Dirac fermions. On the other hand, optics has offered on many occasions a test bed to investigate the dynamical aspects embodied in a wide variety of coherent quantum phenomena (see, for instance, [9] and references therein). In optics, several proposals of KT analogs have been suggested as well, including light propagation in deformed honeycomb photonic lattices [10] whose band structure is similar to the one of graphene [11, 12], light refraction at the interface between positive-index and negative-index media [13], spatial light propagation in binary waveguide arrays [14], and stationary light pulses in an atomic ensemble with electromagnetically induced transparency [15]. The experimental implementations of such schemes, however, might be a nontrivial matter, and an experimental observation of KT for photons is still lacking. On the other hand, multilayer and Bragg dielectric structures, such as fiber Bragg gratings (FBGs), are rather simple photonic devices with flexible design that have been successfully demonstrated to provide an accessible laboratory tool to investigate photonic analogues of non-relativistic tunneling phenomena [16–18]. Here it is shown that an optical analogue of KT can be achieved in a nonuniform FBG composed by

two periodic sections linked by a chirped section which mimics an external potential step in the Dirac equation. Such a FBG-based system might be considered the simplest system proposed so far in order to observe Klein tunneling in any optical system.

II. QUANTUM-OPTICAL ANALOGY

The starting point of our analysis is provided by a standard model of light propagation in a FBG with a longitudinal refractive index $n(z') = n_0 + \Delta n m(z') \cos[2\pi z'/\Lambda + 2\phi(z')]$, where n_0 is the effective mode index in absence of the grating, $\Delta n \ll n_0$ is the peak index change of the grating, Λ is the nominal period of the grating defining the reference frequency $\omega_B = \pi c/(\Lambda n_0)$ of Bragg scattering, c is the speed of light in vacuum, and $m(z')$, $2\phi(z')$ describe the slow variation, as compared to the scale of Λ , of normalized amplitude and phase, respectively, of the index modulation. Note that the local spatial frequency of the grating is $k(z') = 2\pi/\Lambda + 2(d\phi/dz')$, so that the local chirp rate is $C = dk/dz' = 2(d^2\phi/dz'^2)$. The periodic index modulation leads to Bragg scattering between two counterpropagating waves at frequencies close to ω_B . By letting $E(z', t) = \varphi_1(z', t) \exp[-i\omega_B t + ik_B z' + i\phi(z')] + \varphi_2(z', t) \exp[-i\omega_B t - ik_B z' - i\phi(z')] + c.c.$ for the electric field in the fiber, where $k_B = \pi/\Lambda$, the envelopes φ_1 and φ_2 of counterpropagating waves satisfy the coupled-mode equations [19]

$$i [\partial_{z'} + (1/v_g)\partial_t] \varphi_1 = (d\phi/dz')\varphi_1 - \kappa(z')\varphi_2 \quad (1)$$

$$i [-\partial_{z'} + (1/v_g)\partial_t] \varphi_2 = (d\phi/dz')\varphi_2 - \kappa(z')\varphi_1 \quad (2)$$

where $\kappa(z') \equiv [k_B m(z') \Delta n]/(2n_0)$ and $v_g \sim c/n_0$ is the group velocity at the Bragg frequency. The analogy between pulse propagation in the FBG and the Dirac equation in presence of an electrostatic field is at best captured by introducing the dimensionless variables $z = z'/Z$ and $\tau = t/T$, with characteristic spatial and time scales $Z = 2n_0/(k_B \Delta n)$ and $T = Z/v_g$, and the new envelopes $\psi_{1,2}(z') = [\varphi_1(z') \mp \varphi_2(z')]/\sqrt{2}$. In this way, Eqs.(1-2) can be cast in the Dirac form

$$i\partial_\tau \psi = -i\sigma_1 \partial_z \psi + m\sigma_3 \psi + V(z)\psi \quad (3)$$

for the spinor wave function $\psi = (\psi_1, \psi_2)^T$, where $V(z) = (d\phi/dz)$ and $\sigma_{1,3}$ are the Pauli matrices, defined by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (4)$$

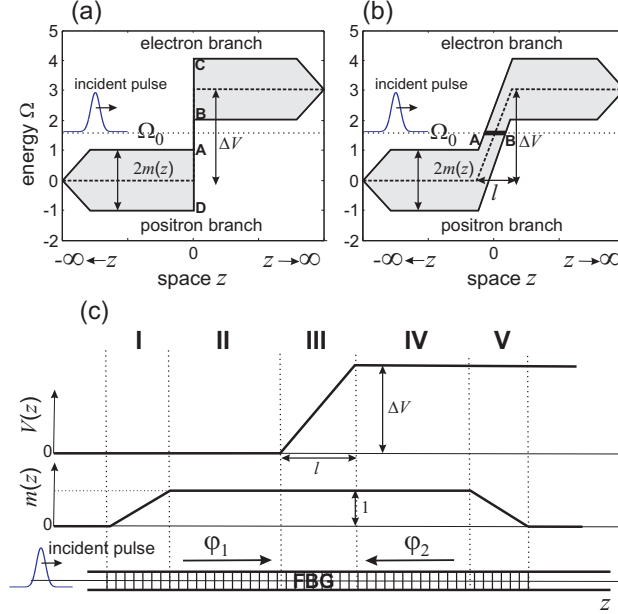


FIG. 1: Energy diagrams of the Dirac equation (3) for (a) a sharp, and (b) a smooth potential step $V(z)$ of height ΔV . The shaded regions are the forbidden energies that separate the electron and positron states, the dotted horizontal line is the energy Ω_0 of the incoming wave packet, and the dashed curve is the shape of the potential step $V(z)$. (c) Schematic of the grating structure that realizes the optical analogue of relativistic tunneling across a potential step. The grating comprises five sections, denoted by roman numbers (from I to V), that are defined by the amplitude $m(z)$ and phase gradient $V(z) = (d\phi/dz)$ grating profiles.

In its present form, Eq.(3) is formally analogous to the one-dimensional Dirac equation with $\hbar = c = 1$ in presence of an external electrostatic potential $V(z)$, m playing the role of a dimensionless (and generally space-dependent) rest mass (see, for instance, [2, 4]). As is well-known, a nonvanishing mass m is responsible for the existence of a forbidden energy region, which separates the positive- and negative-energy branches of the massive Dirac equation. The optical analogue of the forbidden energy region is precisely the photonic stop band of the periodic grating. As the refractive index modulation of the grating, i.e. the mass term m in the Dirac equation (3), is decreased, the stop band region shrinks and the limit of a

massless Dirac equation (similar to the one describing the dynamics of electrons in graphene near a Dirac point) is attained. The additional external potential V in Eq.(3), related to the chirp of the grating according to $V(z) = (d\phi/dz)$, changes the local position of the forbidden energy region. Therefore, pulse propagation in a FBG with a suitably designed chirp profile can be used to mimic the relativistic tunneling of a wave packet in a potential step $V(z)$. It should be noted that, as compared to other photonic analogues of KT recently proposed in Refs.[10, 14] and based on *spatial* light propagation in periodic photonic structures, the phenomenon of KT occurring in FBGs and discussed in the following section involves the *temporal* (rather than the spatial) light dynamics and can be therefore simply investigated in the frequency domain by spectrally-resolved transmission measurements.

III. KLEIN TUNNELING

To realize the analogue of KT, let us first assume that the optical pulse propagates in a region of the grating where $m(z)$ is uniform and equal to one, and let us assume a chirp profile that mimics a step potential with an increase from $V = 0$ to $V = \Delta V$ which occurs over a length l (see Fig.1). Since for the Dirac equation (3) written in dimensionless units the Compton length is $\lambda_C = 1$ and the rest energy is $mc^2 = 1$, according to Sauter's analysis KT is expected to be observable for l smaller than ~ 1 and for a potential height ΔV larger than 2 [2–4]. The process of KT and tunneling inhibition for a smooth potential step can be simply explained by a graphical analysis of the space-energy diagrams (z, Ω) of the one-dimensional Dirac equation [2], which are shown in Figs.1(a) and (b) for a sharp and for a smooth potential step, respectively. For the sake of clearness, in the figures the potential $V(z)$ has been chosen to yield a nonvanishing and constant chirp rate over a length l ; different forms for the potential step, such as the profile $V(z) = (\Delta V/2)[1 + \tanh(z/l)]$ considered in the seminal work by Sauter [3], can be assumed as well without changing the main results. The space-energy diagrams of Figs.1(a) and (b) schematically show the behavior of the energy spectrum of Eq.(3) versus z , which is composed by two branches -the electron and positron energy branches of the Dirac equation- separated by a gap of width $2m(z)$ and centered along the curve $\Omega = V(z)$. The gap regions are visualized in the diagrams by the shaded areas. A wave packet (optical pulse) in the electron branch with an initial mean energy Ω_0 ($1 < \Omega_0 < \Delta V - 1$) coming from $z \rightarrow -\infty$ tunnels into the

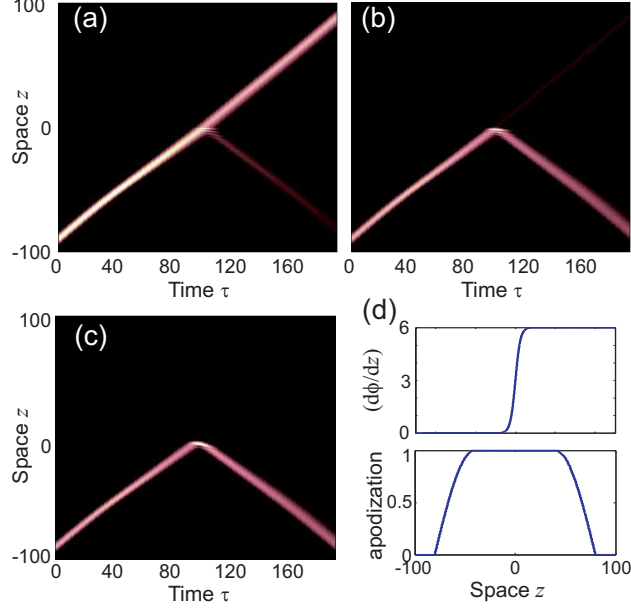


FIG. 2: (a-c) Pulse propagation in a FBG with a chirp profile $V(z) = (\Delta V/2)[1 + \tanh(z/l)]$ for $\Delta V = 6$ and for: (a) $l = 0.1$, (b) $l = 1.5$, and (c) $l = 5$. (d) Profiles of $V = (d\phi/dz)$ (upper plot) and of grating amplitude $m(z)$ (apodization profile, lower plot). The potential V is shown for $l = 5$, corresponding to the simulation of Fig.2(c).

$z > 0$ region after crossing a forbidden energy region, indicated by the bold segment AB in Fig.1(b), which vanishes for a sharp potential step [$l = 0$, see Fig.1(a)]. According to Sauter's analysis [2, 3], the tunneling probability is appreciable provided that l is smaller than ~ 1 . In the FBG context, the energy diagrams of Fig.1 are equivalent to the band-reflection diagrams introduced by Poladian for a graphical analysis of nonuniform gratings [20], where the energy Ω represents the frequency detuning of the incoming wave from the Bragg frequency ω_B . The Sauter's condition $l < \sim 1$ for KT can be derived following the analysis of Ref.[20] by computation of the transmittance of the effective grating associated to the evanescent region AB shown in Fig.1(b) (see Sec.V.A of Ref.[20]). In the previous discussion, we assumed $m(z) = 1$, however for a grating with finite spatial extent one has $m(z) \rightarrow 0$ as $z \rightarrow \pm\infty$. To inject and to eject the optical pulse into the $m(z) = 1$ grating region around $z = 0$, an input and an output apodization sections can be introduced, which adiabatically convert the input and output wave packets from the $m(z) = 0$ regions into the $m(z) = 1$ grating region (see Fig.1). Therefore, the general structure of the FBG that realizes a photonic analogue of relativistic tunneling across a potential step consists of five

sections, as shown in Fig.1(c): two boundary apodization sections (regions I and V), and two uniform sections (regions II and IV) separated by a central chirped section of length $\sim l$ (region III). In Figs. 2(a-c) typical examples of pulse tunneling across the potential step $V(z) = (\Delta V/2)[1 + \tanh(z/l)]$ are presented, showing KT for a sharp potential step [Fig.2(a)] and inhibition of tunneling as the step gets smooth [Fig.2(b) and (c)]. The figures depict the temporal evolution of $|\psi_1|^2 + |\psi_2|^2 = |\varphi_1|^2 + |\varphi_2|^2$ -which is proportional to the field intensity averaged in time over a few optical cycles and in space over a few wavelengths- as obtained by numerical analysis of Eqs.(1) and (2) for a grating length of $z = 160$ with a quarter-cosine apodization profile [see Fig.2(d)], $\Delta V = 6$, and for a few values of l . A forward-propagating Gaussian pulse φ_1 of mean energy $\Omega_0 = 2$ coming from $z \rightarrow -\infty$ and of duration (FWHM in intensity) $\tau_p = 5$ has been assumed as an initial condition. For typical parameter values $n_0 = 1.45$, $\Delta n = 3.3 \times 10^{-4}$ and $\lambda_B \equiv 2n_0\Lambda = 1560$ nm, which apply to FBGs used in optical communications, the spatial and temporal scales in Fig.2 are $Z \simeq 1.5$ mm and $T \simeq 7.3$ ps, respectively. Hence, in physical units the grating length is $L \simeq 24$ cm, whereas the optical analogue of the Compton length is $\lambda_C = Z \simeq 1.5$ mm. Such nonuniform FBG structures should be realizable with current FBG technology based on UV continuous laser writing [21]. It should be finally noticed that, as in an experiment the tracing of pulse evolution in the grating (Fig.2) can be a nontrivial task, the signatures of KT can be simply obtained from standard spectral transmission measurements of the grating. In fact, for a given value of $\Delta V > 2$ and according to the band diagram of Fig.1(a), in the KT regime a transmission window at $1 < \Omega < \Delta V - 1$ [the segment AB in Fig.1(a)], embedded into the two gaps $\Delta V - 1 < \Omega < \Delta V + 1$ and $-1 < \Omega < 1$ [the segments BC and AD in Fig.1(a)] should be observed in the transmission spectrum, the suppression of KT for a smooth potential corresponding to the lowering of such a transmission window. This is clearly shown in Fig.3, where the spectral transmittance of the FBGs corresponding to the simulations of Figs.2(a), (b) and (c) are depicted. Note that, as the length l of the chirped region is increased [from Fig.3(a) to 3(c)], the transmission window embedded in the two adjacent gaps disappears, which is the signature of KT inhibition.

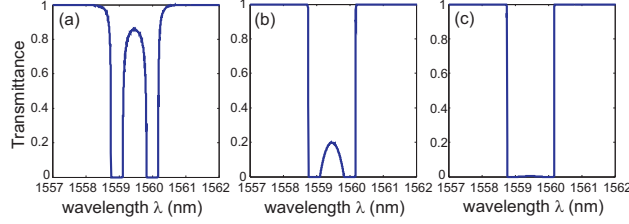


FIG. 3: Numerically computed spectral transmittance of FBGs used in numerical simulations of Fig.2 for $n_0 = 1.45$, $\Delta n = 3.3 \times 10^{-4}$, $\lambda_B = 1560$ nm, corresponding to a grating length $L \simeq 24$ cm, and for increasing values of l : (a) $l = 0.1$, (b) $l = 1.5$, and (c) $l = 5$.

IV. CONCLUSIONS

In conclusion, a photonic analogue of Klein tunneling based on pulse propagation in nonuniform fiber Bragg gratings has been proposed. As compared to other photonic analogues of KT recently proposed in Refs. [10, 14] and based on spatial light propagation in periodic photonic structures, the phenomenon of KT in FBGs suggested in this work can be simply observed in the frequency domain as the opening of a transmission window inside the grating stop band, provided that the impressed chirp is realized over a length of the order of the analogue of the Compton wavelength. Such a FBG-based system might be thus considered to be the simplest optical analogue proposed so far to observe KT.

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